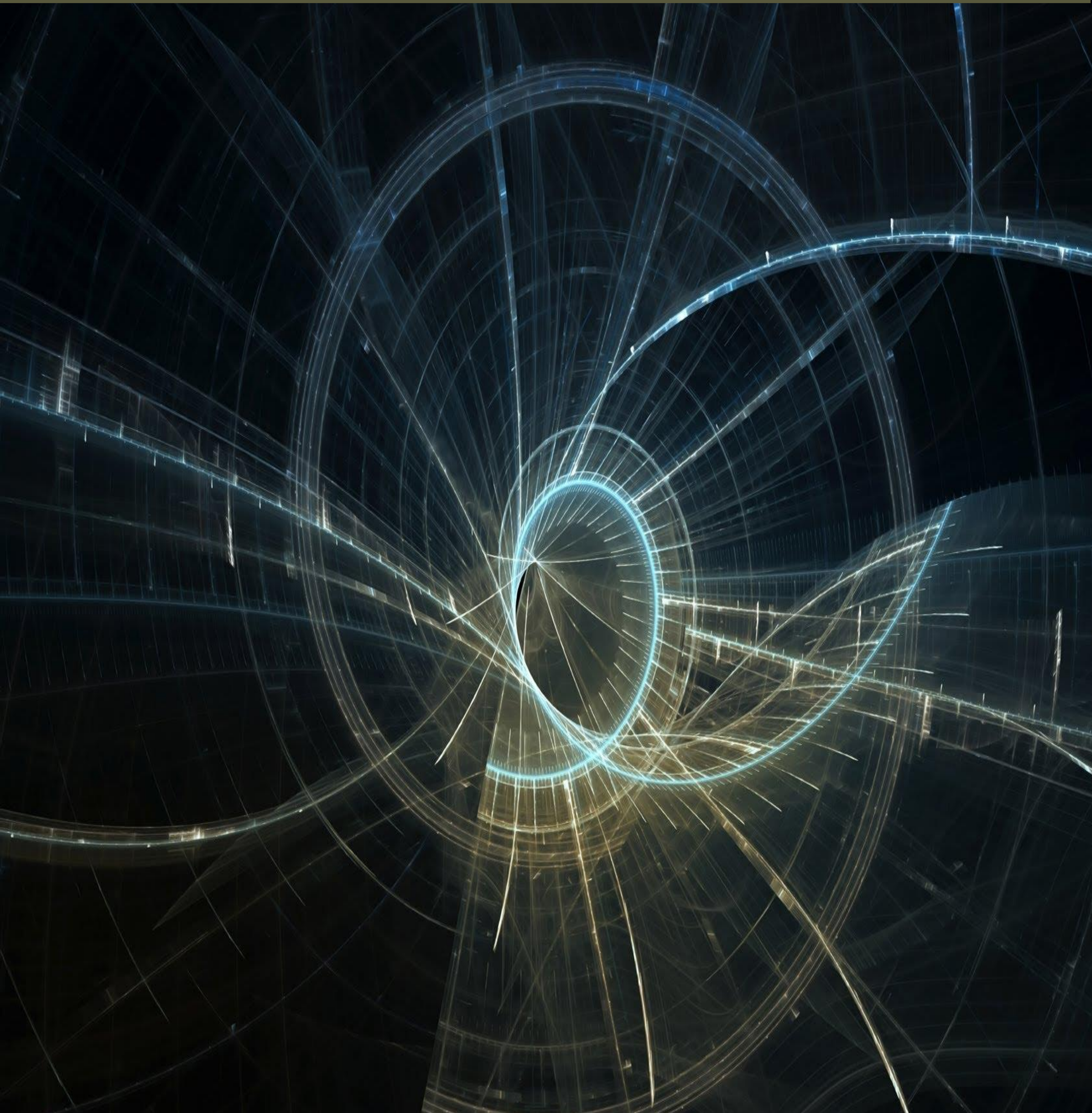


METACOGNITIVE SCAFFOLDING

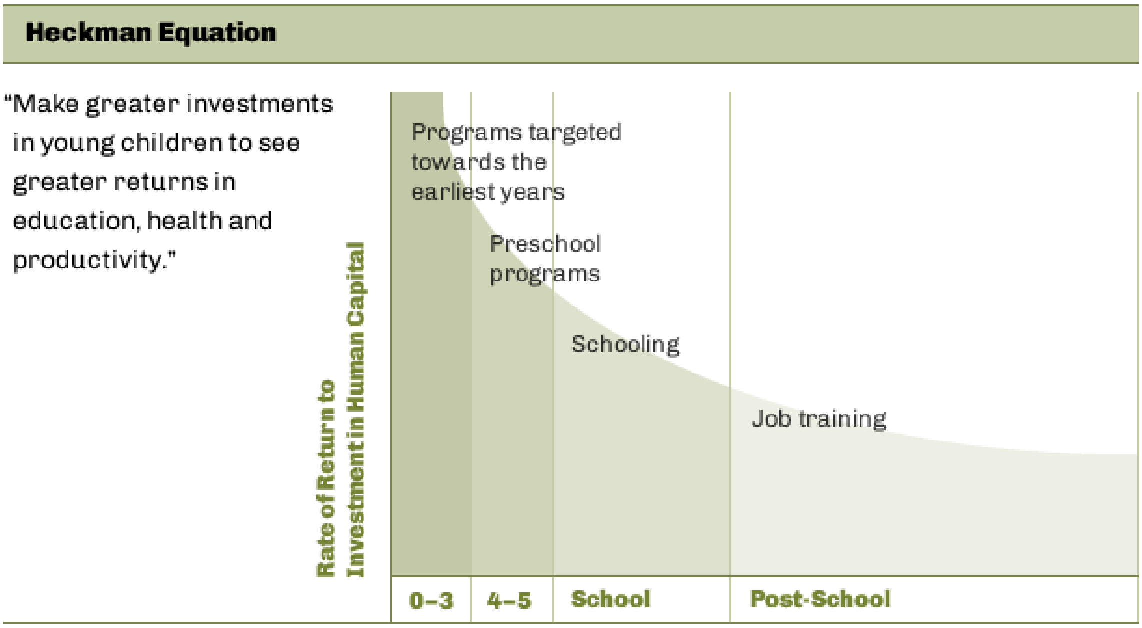
WITH QUANTUM-LIKE MODELS

Introduction

The Nobel Prize winner James Heckman has demonstrated through the analysis of statistical data in the U.S.A. that the first years of childhood are vital for the formation of cognitive abilities such as empathy, motivation and self-esteem. Their interaction with gross intelligence, which begins to form in the Primary stage, allows for the later acquisition of work skills, whilst also leaving a deep imprint on the people who come to condition their personal and professional development later on.

It was found that a very great proportion of the wage differences in adults came from the lack of development of non-cognitive abilities in childhood, from having grown up in problematic family environments or simply from receiving an inadequate and insufficient education at the Primary stage. This irreversibly affected future employability and human development in the long-term.

What is now known for certain is that educational programs that aim at reducing inequality and providing equal opportunities should focus their resources on caring for disadvantaged children at a very early age. The cost-effectiveness ratio surpasses by far its alternatives.



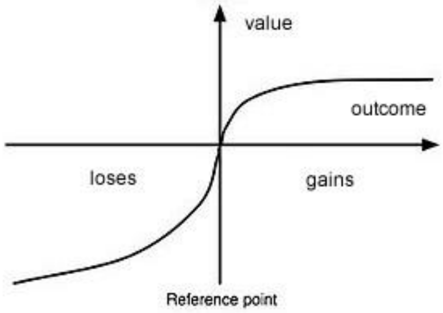
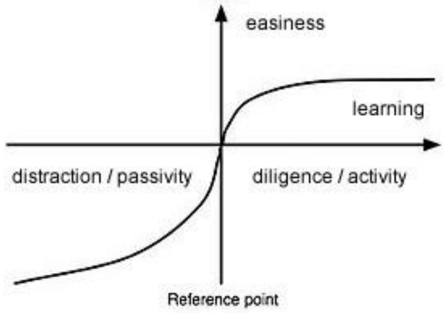
Furthermore the Nobel prize-winner Daniel Kahneman demonstrated along with Amos Tversky that individuals often make decisions in uncertain environments contradicting basic principles of probability. This relates to the idea that in situations where the information available is very complex, excessive or insufficient, actions end up being decided through what is called `heuristic shortcuts' because they save on mental resources.

It is through this system that we are able to compensate for specific errors with multiple subsequent successes. That is to say, decisions end up being made by a contextualizing economy within an established framework of prejudices of rational appearance, but not infrequently these prejudices turn out to be deeply irrational when subjected to quantitative analysis.

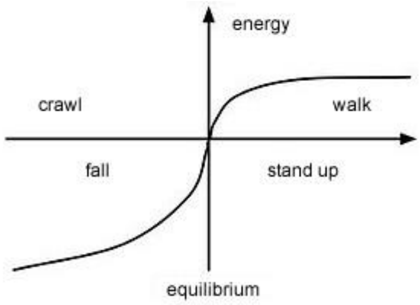
The theory that analyzes the form in which individuals value losses and gains is called 'prospect theory'. This theory describes the decision-making process in two stages, editing and evaluation. In the first stage, the results are analyzed following a heuristic schema centered on a reference point that orders the *subjective* value of outcome benefits. It is at this moment the second stage commences, where people behave as if they can compute a value (utility), based on the potential outcomes and their respective probabilities, and then choose the option that has a higher utility.

Empirical data collected in testing carried out in recent years in various educational centers has allowed for the establishment of two conclusions. The first is demonstrable, the second is only a hypothesis pending demonstration:

1. The heuristic schema discovered by Kahneman and Tversky has a comparative schema in the learning processes as shown in the following diagram:

	Decision	**Learning**
Easiness	Heuristic	Spontaneity
Difficulty	Evaluation	Effort
Heuristic process		

2. The heuristic model may affect all later learning as it is the first complex knowledge schema that humans obtain by themselves, having as its origin the discovery of the balance and the corresponding abandonment of crawling in favor of walking.



Without going into the evidence, the following are the 'reference point' principles:

1. Each student has an established point of reference, in regards to their cognitive ability.
2. The closer the teachings are to this reference point, the more quickly the learnings are consolidated.
3. This is related to intelligence profiles and these profiles in turn can be grouped according to the student’s cognitive process.
4. The reference point can be revealed through a metacognitive analysis.
5. Quantum models encounter probability patterns to reveal it.
6. When the student discovers how to learn better, learning improves fast because they also learn to approach difficulties in a self-sufficient way to facilitate comprehension.

In the learning process, beyond what is actual learned, the very idea of learning itself is discovered. That is to say, in the same cognitive process, meta-cognitive aspects are revealed that are related with the knowledge of a well-known object. Or, to say it in another way, metacognitive awareness is discovered by analyzing the very same knowledge.

We possess therefore an implicit understanding of our knowledge, although in an imperfect form. To express this in a more simple way, nobody realizes what they know until they know it. But that implicit knowledge is difficult to perceive due to the ease with which it is confused with the well-known. It behaves just like a good actor who disappears in front of an audience, becoming simply a representative of a character.

With regards to the teaching process, the order with which to discover it is as follows:

1. Teacher Explanation.
2. Student conceptualization.
3. Learning.
4. Knowledge of the learning.
5. Learning of the learning.

Therefore, the knowledge that the student obtains is not only due to the teacher explanation, but also it has an internal locutive spontaneous action that encompasses the very act of knowing. It is in this second intellectual or metacognitive level that is where the master key is located and where the teacher finds the educational content (sequence of exercises in the case of mathematics) most suitable for the student, whilst at the same time the student also finds the intellectual strategy most suitable to accelerate his understanding and later learning.

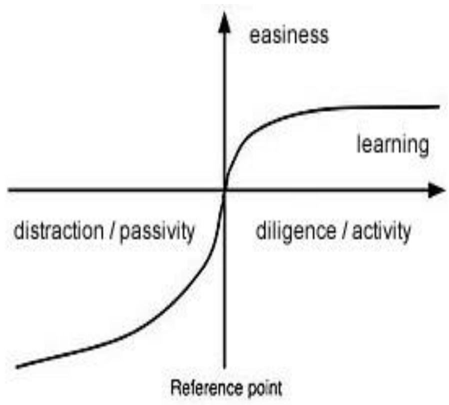
When the student discovers that at the same time that they are understanding and learning the explanations, they are understanding and learning their own understanding and learning processes, it automatically sets in motion a new strategy to improve self-cognition, similar to what occurred in that first year of life when crawling was left behind in favor of autonomous mobility.

With these explanations it is evident that knowledge involves more than mere passive reception. When the teacher is able to share information in such a way that the students discover and perfect their own intellectual mechanisms, they are then able at the same time to initiate an acceleration of the learning process through the same discoveries that the student is making. Therefore, the best collaboration that the students can enter into in class is not so much to put forth an intellectual effort to incorporate the explanation of the professor (evidently necessary), but rather to deepen the form in which they know and they learn. For that it is how each one of them can optimize their ability to continually improve, until they are able to accommodate their intelligence profile to the subject matter, independent to the difficulties that they will encounter and the effort required to overcome them.

To understand is therefore not only the possession of what is understood, but also an internal locutive act that manifests the same understanding. And as understanding something is not something distinct from the act of understanding, students learn that they can be taught without great effort being added to their own learning process. **Once metacognition is included as something habitual in the classroom, mathematical problems (or those related to any other subject) that are strictly metacognitive are the natural evolution, as they are the next step in the metacognitive scaffolding**.

It is very important to stress that students have the ability to improve their own intellectual process, exactly as they also learned to walk alone. It was Aristotle himself who discovered this fact against the immobility of Parmenides, reducing it to a principle of contradiction by denying the absurd:

"Who denies the movement affirms it, because to think it is to move [by syllogisms]"



Going back to Kahneman, the improvement of the learning process takes place within the same learning process, because the ease/difficulty that the student encounters bears relation to a reference point limit from which the new knowledge becomes simpler. This 'reference point' can be standardized by the individual biases of the intellectual abilities, according to the intelligence profiles. Thus according to Kahneman and Tversky, although the learning process is costly in terms of time and effort, by and large the process produces and advances thanks to 'intellectual short cuts' that facilitate the process.

In this way it is possible to have children with ease or ability in some subjects, but difficulty in others. They have an improved ability to establish 'intellectual short cuts' only under very specific cognitive environments. **The aim of this new pedagogical approach is simply to provide a powerful analytical tool for the teacher to help in teaching and helping students to find an appropriate level of intelligence to fit their profile easily, thus reducing intellectual effort, as students can learn more easily while at the same time they better assimilate and consolidate the new knowledge acquired**. Experience demonstrates that, in effect, when a student finds their own 'short cuts' it not only increases their motivation, but that the learning is catalyzed by the new metacognitive abilities obtained, which themselves are a superior level of more advanced learning, because the process feeds itself.

Returning to Heckman, taking into account the findings presented here and given the importance of cognitive and non-cognitive skills in younger ages, governments have an overriding obligation to implement these advances so that people can look forward to development in the future, including economically, especially when taking into consideration the new society to which we are heading. A society in which millions of jobs will be replaced by computerized processes and cognitive computer systems.

Inaction is not an option, this is urgent!

QUANTITAS Project

Metacognition and scaffolding

Metacognition

The term metacognition was coined by Flavell (1979) and is usually defined in a simplified way as “the knowledge about the knowledge”, that is to say, knowledge that includes not only your own intellectual process but in addition the cognitive regulation that goes with it. Metacognitive knowledge is therefore personal knowledge regarding the form in which knowledge is created, whilst also being about how those same cognitive abilities are handled in order to monitor, evaluate, revise and solve complex analytical structures.

Scaffolding

Scaffolding refers to the necessary support provided by the teacher or the trained peer tutoring the students while they are as yet unable to progress independently in the learning process.

There are four types of scaffolding:

1. Conceptual: To facilitate access to the complexity of the problems
2. Metacognition: In order to adapt the problem to the intellectual profile of the student, whilst at the same time guiding them in their own meta-cognitive processes
3. Procedural: Regarding the way to use resources and tools
4. Strategic: Guidance in the way to approach problems

If the students are not involved from the very beginning in the basic development of the subject, it will not be possible to introduce much more in terms of metacognitive activities later. Therefore, everything that is related to metacognition and the scaffolding of teaching sequences evidently requires as a minimum a collaborative attitude in the classroom with the students.

There exists little research regarding metacognitive scaffolding with regards to solving deconstructed complex problems. There is even less research into the evolution of the student’s learning process in problem solving depending on their intellectual profile. But research has demonstrated that when teachers prepare pedagogical strategies that include metacognitive elements, the academic results are better as students consolidate abilities related to the representation of complexity, finding solutions, justification of procedures and results and especially the explanatory power to other students through educational processes created by them to transfer their skills to their peers.

Another important aspect of metacognitive teaching is that it helps the teacher to better understand the conceptual necessities of the students and their intellectual profiles. Permanent feedback is established in the educative process.

Metacognitive scaffolding and complex problems

The solving of complex and poorly structured problems is not a linear or simple process. In fact it is impossible by definition to teach students how to solve them following modeled patterns and therefore the only valid tool is to teach the learning of advanced metacognitive skills, to the extent that these skills enable students to cope with uncertainty. This learning is cyclical and iterative, that is to say, it is possible for it to be taught, to be monitored and to be evaluated.

Although there is no absolute consensus on the importance and necessity of students finishing their formative stage being capable of solving these kind of poorly structured or complex problems, it can be considered to be very valuable indeed but by no means simple to obtain. Thus, to be successful in this, a personalized education process is necessary, often in a very exhaustive form. That there are few scientific publications regarding metacognitive scaffolding is proof of the number of obstacles in the way of implementing such a process.

With regards to the exercises, they vary in type based on the structure, context, complexity and specific domain. In the case of problems with stated complexity, they also vary based on external factors like the number of elements, the functions and variables of the problem, the amount of interactions between those elements and the predictability of the behavior of those elements, functions and variables.

Complex problems require more cognitive operations than simple problems and impose a higher cognitive load on the student to be able to solve them. In fact, complexity and structure overlap. For that reason, some badly structured problems can be quite simple in their resolution. In any case, problems with a planned design are some of the most complex and for that reason they require greater metacognitive abilities.

There exists a sufficient variety of problems that require metacognitive abilities, that it is therefore possible to personalize the different student intellectual profiles according to each part of the syllabus. Here reference is mainly made to the subject of mathematics and more specifically to the problems of greater complexity and those that require greater metacognitive abilities. Among them eleven types of problems have been identified:

1. Logical.
2. Algorithmic.
3. Historical.
4. Regulated.
5. Regarding decisions.
6. By anomalies.
7. Diagnostic.
8. Strategic.
9. With situation analysis.
10. By design.
11. With dilemmas.

It should be kept in mind that not all students develop the same kinds of metacognition, so you have to design specific problems for each intelligence profile. Thus it is necessary to enable metacognition based on previously encountered metacognitive capacities. It is only in this way that it becomes possible to lead the student towards that 'reference point' that triggers their own personal 'intellectual short cuts'.

Software: Session Typology/Subjects

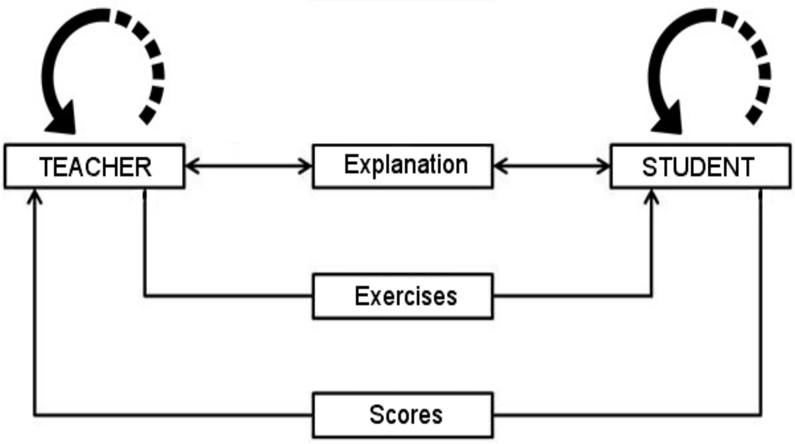
There are three sessions: Class, Home and Training that correspond to the three central applications.

After selecting the sessions, it is possible to choose between the subject and the course. In this way there is maximum flexibility for the teacher with regards to the relationship between any classes and centers.

Finally, the educational sequence comes: There are three types, First, Second and Third.

- The **First** takes place immediately after the explanation of the concepts and corresponds to exercises with increasing but mechanical difficulty.
- The **Second** takes place as a final review before the exam. This second type of educational sequence can occur in the middle or at the end of the unit, according to the understanding of the teacher.
- The **Third** type includes the most complicated concepts in the subject, including the exercises and problems with concepts of greater difficulty and above all the essentially metacognitive problems.

The system is based on understanding the class as an emitting-receiving structure, where the receiver is multiple and each part that intervenes in the communication (education) are conscious subjects. That is to say, they are able to value their learning and to learn from their own cognitive process. To assess this it is possible to carry out metacognitive objective analyses.



Each Class session has a corresponding time allocation, by default this is fifty minutes.

In addition, there is a color indicator (red/green) to recognize if there is already a session open for that class. The option by all means exists to increase the time of the sessions, something especially useful if the mathematics teacher needs to incorporate the following class for unplanned for reasons, due to teacher absence or simply for educational reasons.

The Training session is always open when the other sessions are closed.

The Class session is opened by the teacher and thus the other two are automatically closed, independently of whether they have pending exercises to be solved.

When there are exercises given out, the system leaves the Class open and once finished the students can stay to solve the Training problems. Even so, in the case of not having completed all the exercises, the Class session commands the other two and they are blocked when the teacher opens the classroom. Of course, the exercises sent home for each student include those left over from previous sessions.

Sending exercises, levels of the agenda and generation of data on answers

Sending exercises

Following the above communication scheme, each type (or series of types) explained by the teacher has to be immediately monitored, to get an accurate measurement of the degree of understanding of the class and each student for the answers.

For this purpose, the teacher through the system launches a series of three exercises for each type explained. That is to say, if a series of four types of exercises has been explained during 20 minutes, when the professor sends them to each student, there are twelve exercises.

To each type corresponds a set of exercises with the same difficulty, but with numerical variations. That is to say, the same type of exercise could have 50 variations. In this way there is enough for each student to consolidate their learning, while at the same time in class no one can be copied (which would falsify the data received) because a different exercise will appear for each of them. The system saves which exercises were sent to which student and only repeats the sets of exercises when these have been completely exhausted.

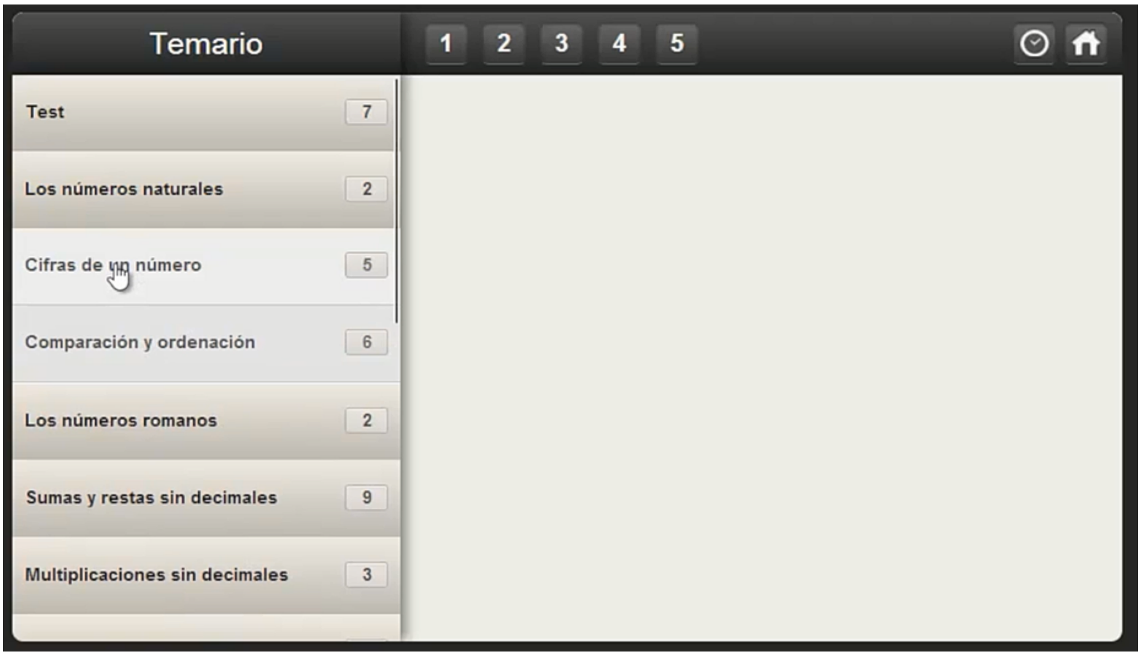
At the same time, each one of the exercises allows for three attempts at the answer, with three well-differentiated grades: 10 (first attempt), 8 (second), 5 (third), 0 (fourth and last attempt). The difference in weight between the first two attempts and the second two attempts was done intentionally in order to divide the statistics into two groups. The reason for this will be explained further ahead.

Once the responses have been produced, the system automatically manages homework, but also allows the teacher to incorporate more or less work at their discretion. A maximum time of 30 minutes of homework is calculated. Normally the system usually sends homework to a third of the class on average, with the aim of strengthening or finishing exercises for those students who have shown to experience more difficulties. The system automatically programs homework as well as other classroom information (grades, class times and exercises) which each student can complete in their own time. And this is both for the teacher's agenda, as well as for each student, and constitutes a summary for parents or guardians.

Levels of the agenda/syllabus

The syllabus has a common standard layout, although at the start of the course the teacher can remove items as necessary. It is organized in three levels, with the third in the form of sequenced typology. Thus it is possible to incorporate any type of exercise (no-CUN or CUN) and all possible teaching sequences according to how each teacher sees fit.

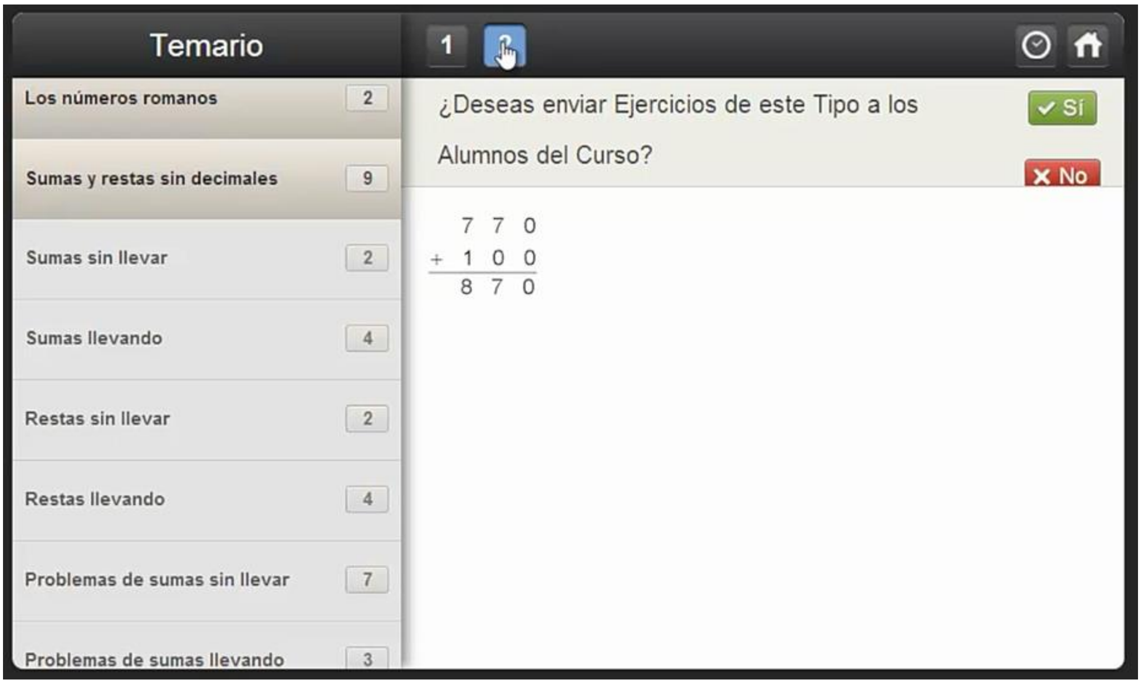
Remember that the exercises are sent through the intervention of the teacher in the Class session, but it is the system that automatically sends to House or Training, cascaded after the class session. Even so, the complete collection of exercises is always available for the students in Training.



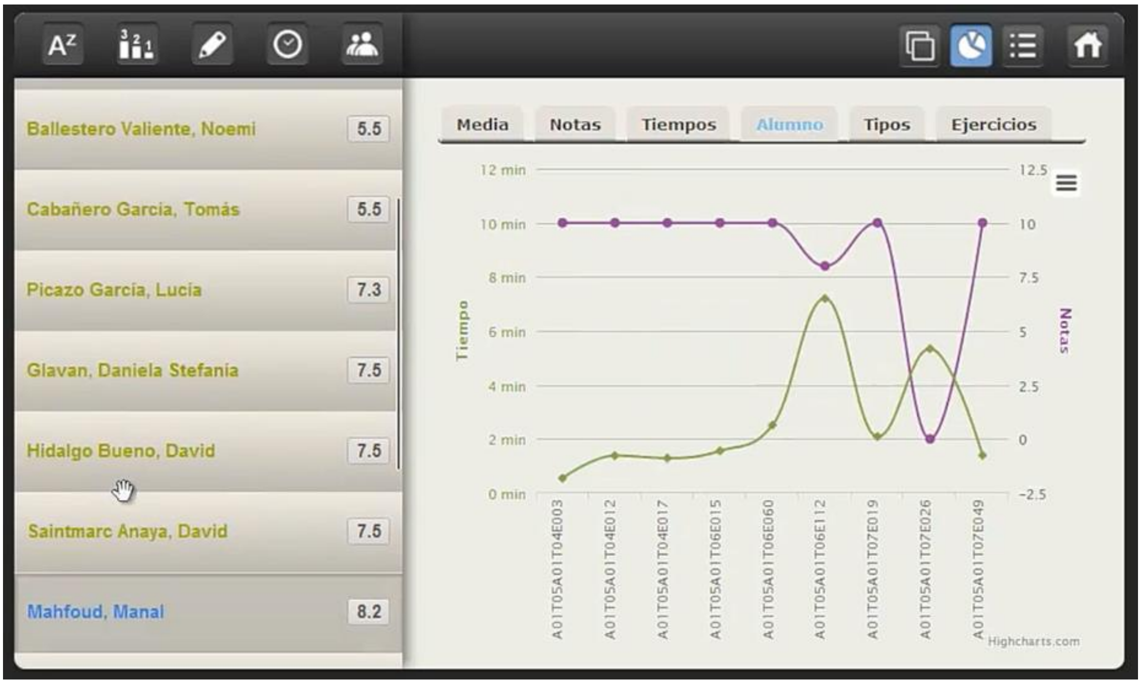
Data generation on answers

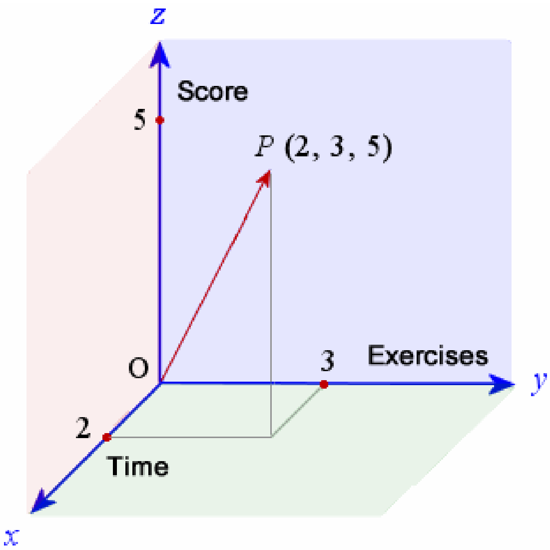
The exercises are presented in the most austere possible format. This is done to keep distraction at a minimum as response time is counted. This is fundamental, as we will soon see.

The student must solve each exercise in the notebook and only input the solution into the device. This system is powered by optimizing the answer process by the mental calculation of the students. With regards to the answer format, multiple solutions are allowed, test style or by selection of fields. In particular, the latter can greatly facilitate the approach to meta-cognitive problems.



The data that is currently collected is: Completed exercises, Grade and Time

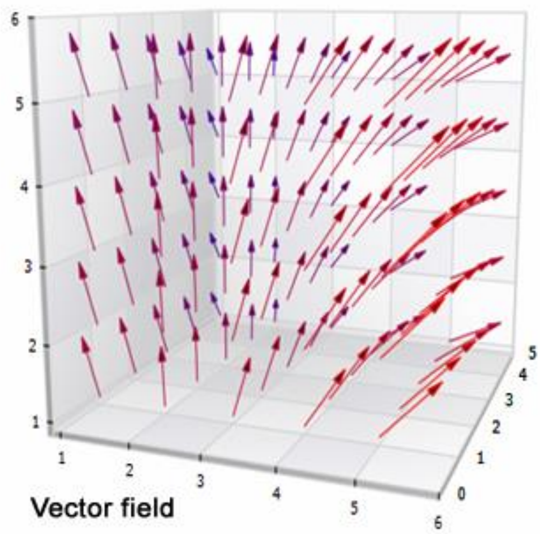




The answer as a vector

What Quantitas achieves is the the vectorization of each student's response in 3D (time, score and number of solutions), for each exercise in the teaching sequence posed by the teacher. By including the time, it is possible to simultaneously analyse the important sequences of attention-distraction and activity-passivity. The study of these two sequences is very important, because it allows for all the later analyses.

The didactic vectorial field



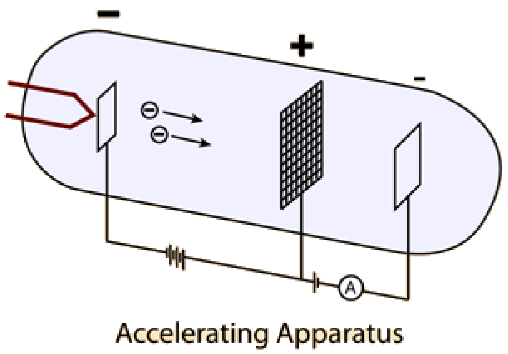
In the teacher-student (s) communication diagram previously mentioned, it is the set of vectors that come from the didactic sequence of the teacher. This field can be measured of three ways:

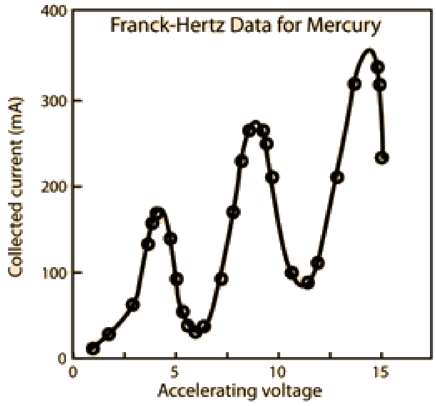
1. In all the answers from one student
2. In one answer for all the students.
3. In all the answers by all the students

First component: Time as a superposition of quantum states

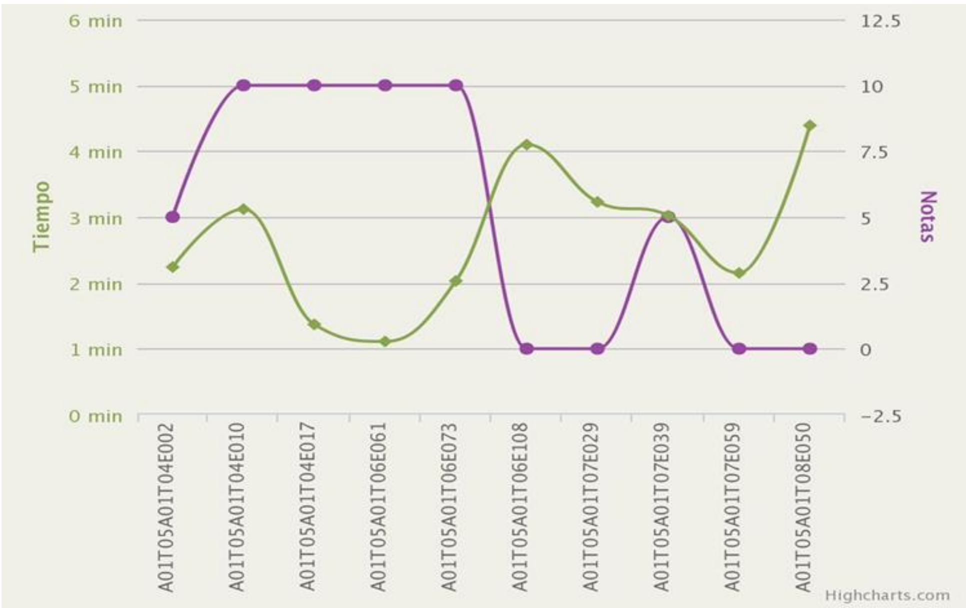
Once it has been explained how the score of the acquired knowledge of the student is reduced to a vector and teaching to a vector field, we go on to explain the first component, time.

In 1913 Niels Bohr proposed his atomic model which was analogous to the planetary system, with the exceptionality of the orbits that should be quantified. It was a theoretical postulate, but it allowed for the explaination of why electrons do not precipitate on the atomic nucleus. In 1914 Frank and Hertz demonstrated this quantization of the orbits finding the levels of energy of the mercury atom.





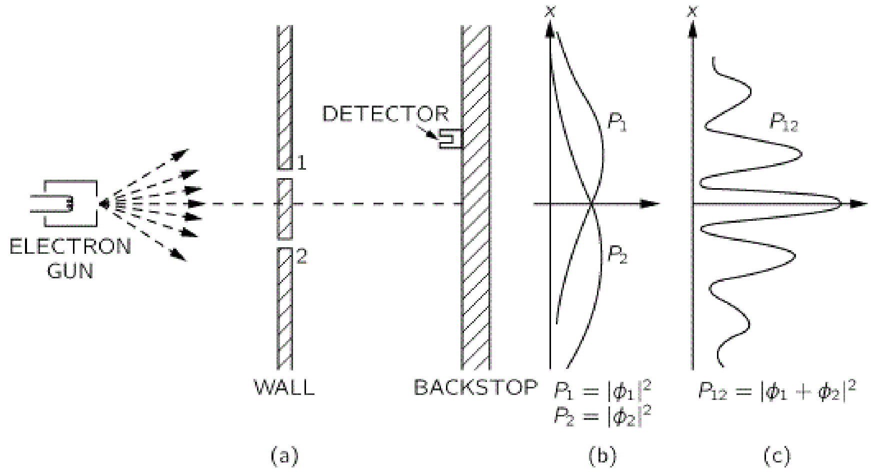
The cognitive hypothesis in Quantitas is analogous, because it estimates that the processes of understanding in the students are ‘quantified in ranks’ according to intelligence profiles. This is seen as temporary variations on the answers to the exercises immediately after the teacher's explanations are analyzed, to the extent that students find the 'benchmark' that facilitates learning. Here the data proving it.



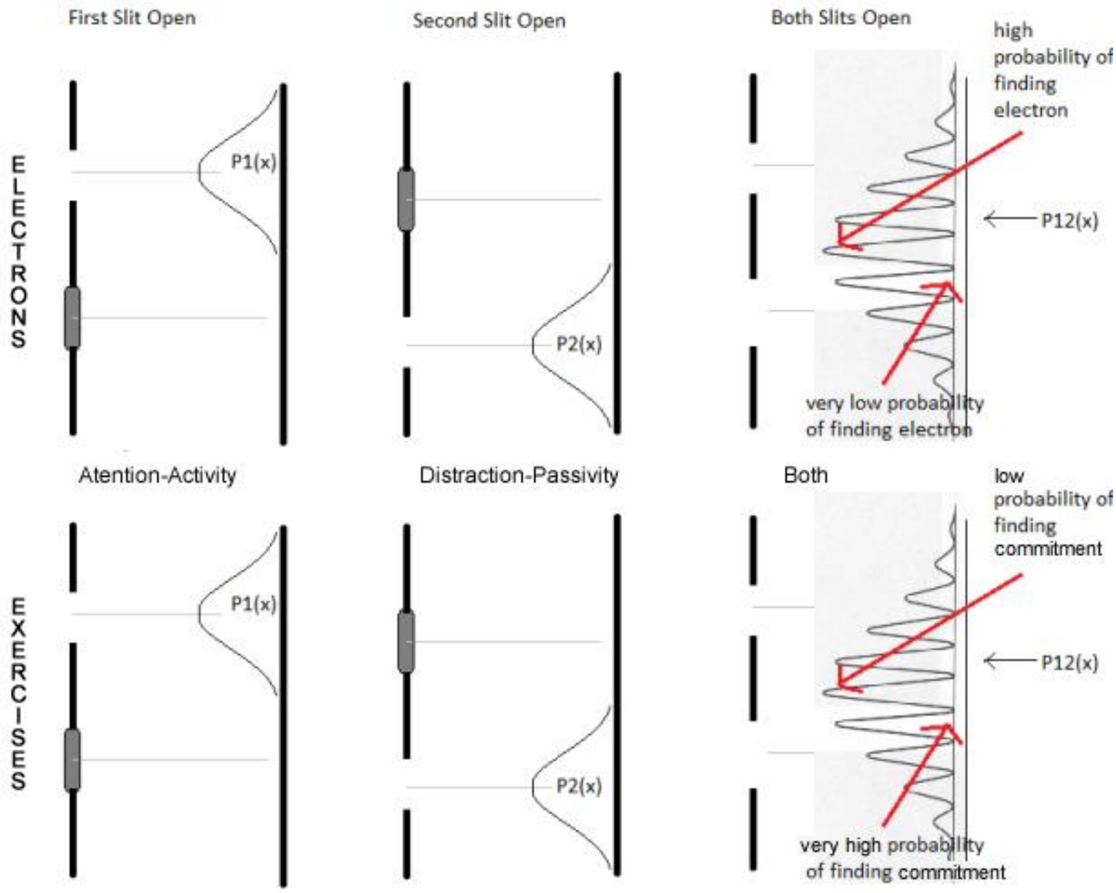
Green color is the time (left axis), violet is the score (right axis). The amount and type of exercises performed is in the bottom axis. This graph is representing a 3D vector analysis. The proof is that when the metacognitive vectorization of each student is shown in class, the grade improves a lot and quickly. This is because students tend (by following the same analogy) to stabilize in a range of focus near their “benchmark”. Look at the difference, with the same student.

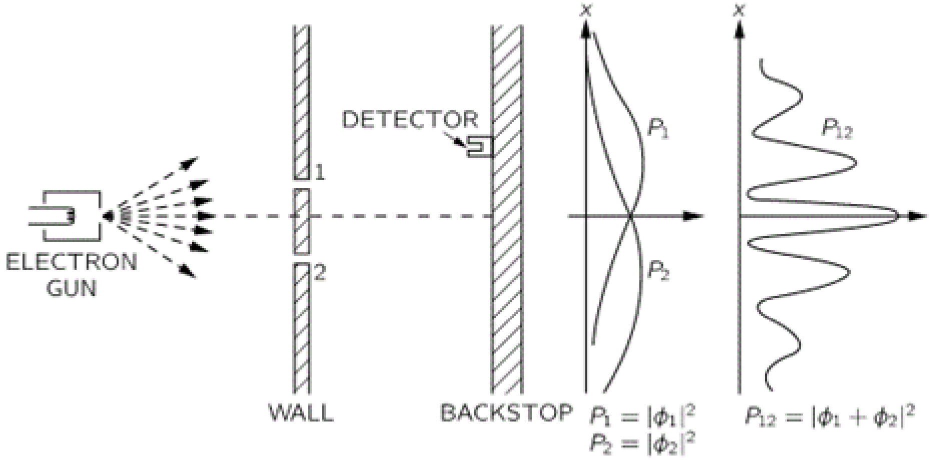
In the tests made, after the class ended, the students are interested by the grade. They all see each other's results and like that they can establish comparative assessments on their own. The display is only used to influence the learning in positive aspects. This way the students assimilate very well the new paradigm of analysis.

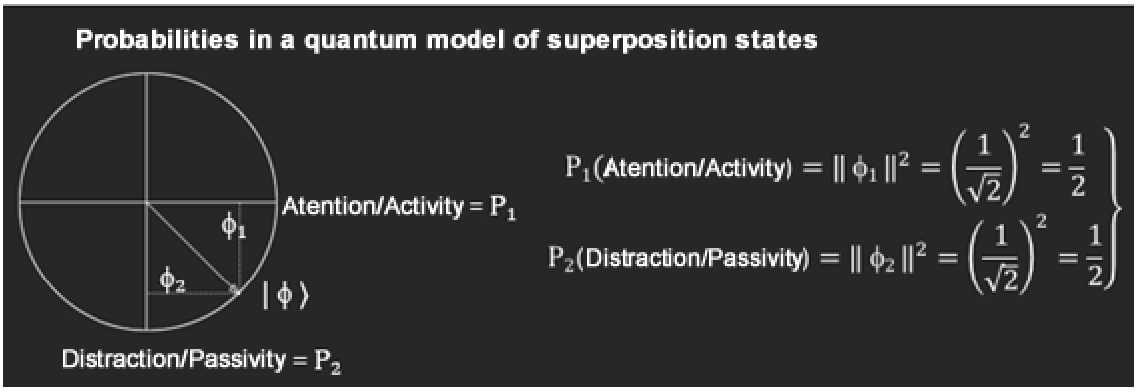
The cognitive 'quantization' process is understood as the result of the natural interference between the attention-activity and the distraction-passiveness during the solving of the exercises. This is what is observed when the solving time is analyzed, that each student has their own 'benchmark' of attention-distraction and activity-passiveness rhythm. When a lot of students are processed with massive data could be established common areas in each type of exercise and in the (quantified) teaching sets.



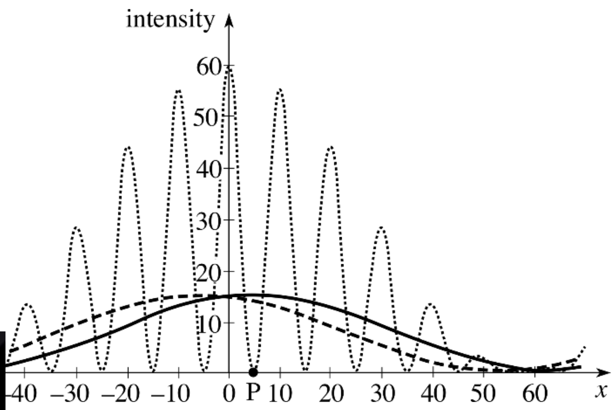
Following the physical analogy, the oscillations can be analyzed as a pattern of interference equivalent to the double slit experiment: the first slit would be the intensity of attention-activity and the second would be the intensity of inattention-passiveness. If we understand both measurements in the function of time, its oscillations are assimilated to those of two waves.







The temporary oscillations can be determined as a quantum interference of two superposed states. To accompany this graph with a visual representation, the classes are recorded at one frame per second, so that a complete class can be displayed in less than 40 seconds.



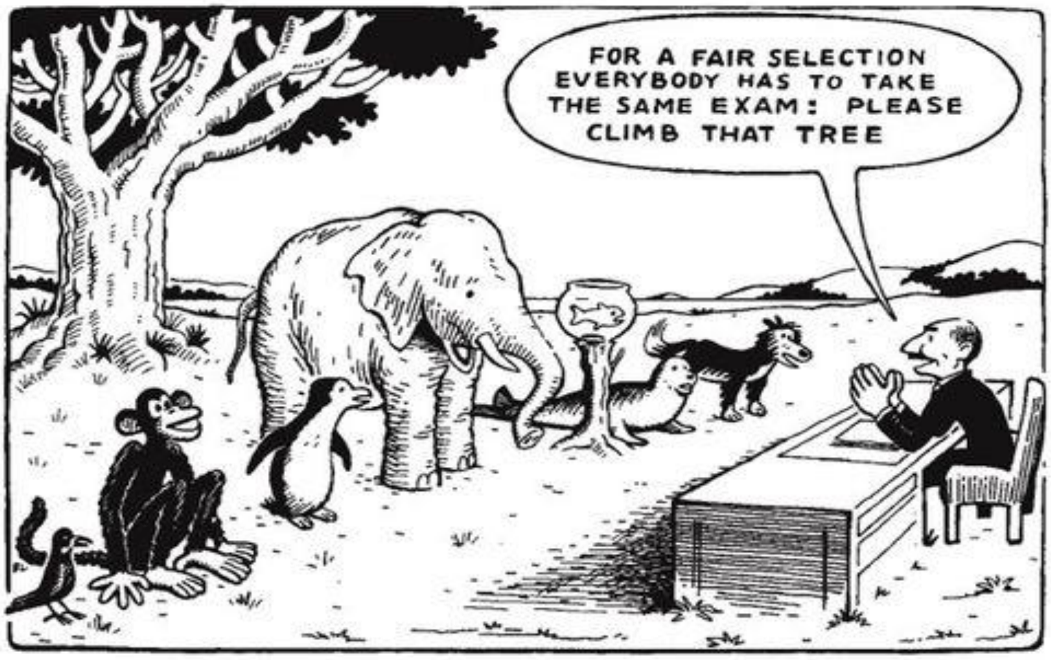
At the same time and to have another receiver of physical mobility, tactile pressure on the tablets is also introduced. This way is how the mentioned oscillations can relate with the parameters of thin and wide motor skills. This is only done in the tests. Later on, the mathematical models have to be precise enough to not need them.

Second element: The grade as an egalitarian fallacy, revised with a quantum-like model

When part of the material of a subject is graded with a test, the grades can be a fallacy against the effort required to pass it. It is possible to have students that score 5.5 and needed 20 hours of extra work at home, while others can get 8.5 just by attending the class and practically with no homework.

In such cases the fallacy manifests negatively, because low grade needs more work than a higher grade but has a lower mark. This means that the grade is not a good measure for evaluating the effort required to pass the test, because it only assesses the accuracy of the answers.

Thanks to the time measurement aforementioned and especially the accuracy that can be obtained in the measurement of attention-activity and distraction-passivity cycles, it is possible to set precise correction patterns that guide teacher's test grades. This does not mean that they should be reducing the brightest students' grades, but what are apparently the worst grades or the barely passing ones on many occasions are the outcome of a great effort that teachers should reward objectively, according to their understanding.

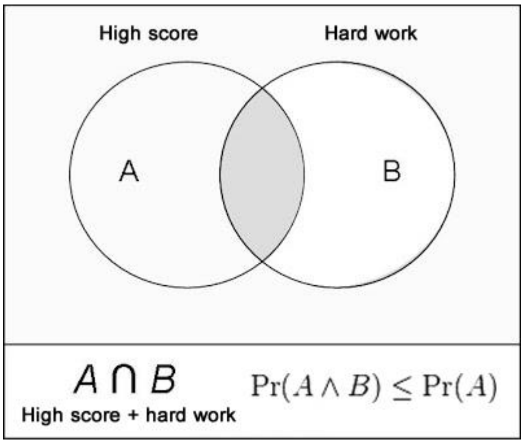


Trying to bring discoveries of Daniel Kahneman (Thinking Fast and Slow, 2011) to the pedagogy inside the classroom, an informal test was proposed to several groups of teachers. The key question was in the middle of the test, and it was as follows:

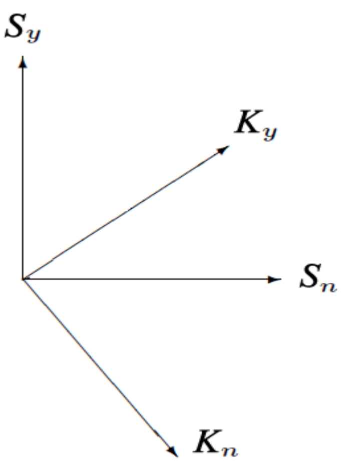
> *In a standard class there is not particularly good study environment. Several board meetings had been focused on trying to find a solution and improve the students' engagement and grades, especially in math. Teachers' opinions were polled regarding the way students' grades could be raised.*
>
> *What is more likely?*
> 1. *A student with good grades*
> 2. *A hard-working student with good grades*
>
> *¿What is your answer?*

Not only did most of the teachers overwhelmingly choose the second option, but they also found it difficult to understand the fallacy that was embodied in their answer, because the cognitive bias related with the 'effort' word. 'Effort' is a restriction on the set of students with good grades, in the same way like 'left-handed' or 'redhead' do. Therefore, the first option is actually more likely. In fact, many students get good grades in subjects by putting in little or no effort thanks to the special ability of their intelligence. Moreover, sometimes there are students who get high grades because they have been lucky enough to be asked about that which they prepared well for. It may also be the case that teachers prepare a very easy test to allow students get good scores. Not to mention those students who get high grades simply because they have been able to cheat the most difficult questions.

Where they all agreed is that the amount of effort does not guarantee good grades. But when teachers were asked how it would be possible to grade effort, the answers had a common denominator of vagueness and the inability to grade effort with objectivity and in fair terms. The graphic correction of the fallacy is displayed in the diagram to the right.



Next, a possible solution will be introduced taking into consideration a quantum-like model, thanks to the quantum-like analysis that has been done on the 3D vector time feature.



Basis, answers YES and NOT

Quantum-Like models are based on questions and answers, analyzing the grade component related to effort like a space vector, introduced to represent the answers "yes" and "no":

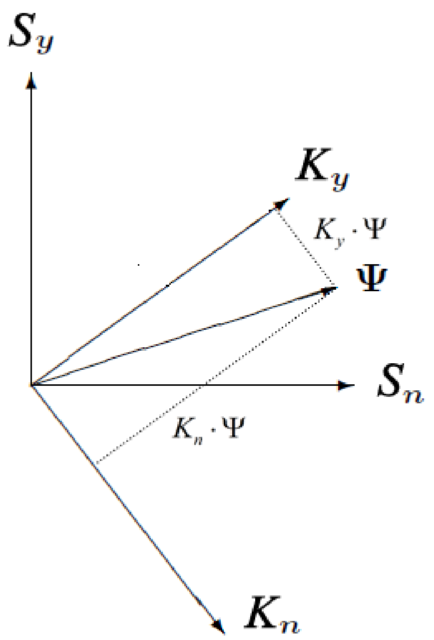
S_y = High score, YES
S_n = High score, NO
K_y = Hard work, YES
K_n = Hard work, NO

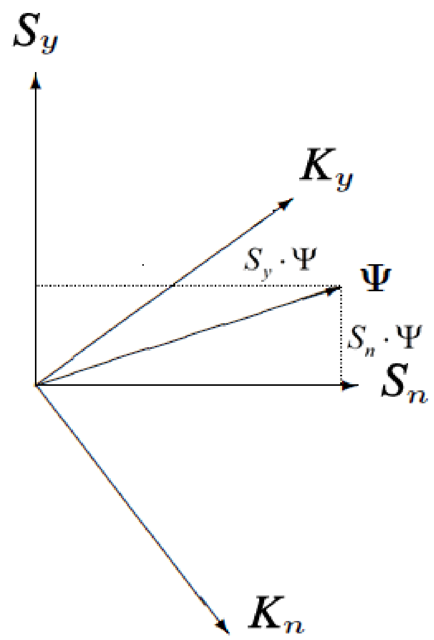
The sets (S_y, S_n) and (K_y, K_n) represent all possible answers to question S and K, and thus are a base for the space vector, which is displayed in the diagram to the left. Note that it is the same vector space that is used to represents answers to questions S and K, both letters are only indicating two different bases.

The vector space is equipped with a scalar product, thus becoming a Hilbert Space: for two vectors $\vec{a}$ and $\vec{b}$, the scalar product $\vec{a} \cdot \vec{b}$ is a complex number. The order of the vectors within a scalar product here matters, that is to say: $\vec{a} \cdot \vec{b} \neq \vec{b} \cdot \vec{a}$.

The bases are orthogonal and of unitary norm:

$$S_y \cdot S_n = K_y \cdot K_n = 0$$
$$S_y \cdot S_y = S_n \cdot S_n = K_y \cdot K_y = K_n \cdot K_n = 1$$





The teacher’s state with respect to the question given is also represented within this Hilbert space, by a normalized vector called Ψ . This vector can be decomposed in either of the two above-mentioned bases:

$$\Psi = (K_y \cdot \Psi) K_y + (K_n \cdot \Psi) K_n = (S_y \cdot \Psi) S_y + (S_n \cdot \Psi) S_n$$

The state of belief Ψ gathers all the relevant information needed to predict the behavior of the teacher. Predictions by quantum-like models are probabilistic. When a question X $(\text{with } X = S \text{ or } K)$ is asked, the probability that the teacher answers i $(\text{with } i = y \text{ or } n)$ is represented by the squared modulus of the scalar product between the state of belief and the vector representing the answer:

$$\Pr(X_i) = |X_i \cdot \Psi|^2$$

This rule is actually called the Born rule, an analogy with the quantum mechanics denomination. Thanks to this rule, one can compute the probability that the teacher will give each of the four answers, in case questions S and K are asked.

One easily checks that $\Pr(X_y) + \Pr(X_n) = 1$, because Ψ is normalized. In the case of a real Hilbert Space, a geometric interpretation of the Born rule is the following: to compute the probability to answer “yes” to question K, orthogonally project Ψ on K_y (this gives the length $K_y \cdot \Psi$). The desired probability is just the squared of this length:

$$\Pr(K_y) = |K_y \cdot \Psi|^2$$

So the more Ψ is aligned with a basis vector X_i, the larger the probability is that the teacher will answer i if question X is posed. Note the “if question X is posed” part, because quantum-like models the probabilities of answers that are defined only in the context when the corresponding question is posed.

The postulation of the quantum-like model has to do with the way Ψ changes over time:

1. When the agent (teacher or student) doesn't answer a question, Ψ doesn't change. This conveys the fact that the agent's beliefs are not influenced in some way or another. This hypothesis is supposed to be relevant for cases in which the questions are posed to the agent relatively quickly.

2. When the agent answers a question K or S, her knowledge or state of belief changes. If the answer to question X is i, then the new state of belief after giving the answer is

$$\Psi \to \frac{X_i \cdot \Psi}{|X_i \cdot \Psi|} \cdot X_i$$

As the fraction is a complex number, the above equation means that the state of belief or knowledge after the answer X_i is proportional to the vector representing this answer, the X_i. In the case of a real Hilbert space, this means that after answering "yes" to question K, Ψ becomes either K_y or $-K_y$, whatever the state of belief or knowledge was before the question. In other words, after a question X has been posed, the state of belief or knowledge is bound to be along the basis vectors representing its answers.

This is what it simply represents. A state of belief or knowledge can be modified by only changing a word in the question. In this case, if the question had been modified by changing the word "effort" to "left-handed" there would have been no doubt about the probability distribution. But what is noted here is that it not only produces the 'conjunction fallacy' in didactic evaluations but that it also adversely affects the teacher's perception of the work of their students. It should be emphasized once again that it is not criticizing here the obvious effort needed in the learning process (especially in the higher states), but rather how fallacies can occur in didactic valuations.

Classically, an answer to a question is supposed to *reveal* a belief, which is pre-existent to the question, and which is precisely the same before and after the question. But there, this cannot be the case: in general, the state of believe is modified by the fact that the question is posed and answered. The only future change will be the time variable: once a question or exercise is answered, the same correct answer will be given if the same question is posed again, but the time to produce the answer will be reduced.

Now consider the defined conjunction fallacy regarding the effort to get good grades in reference to the just developed quantum model. When a teacher sees the conjunction 'hard worker with good grades', this expression is evaluated as a sequence of projections, which is related to two successive answers and dichotomous questions 'yes-no', in this case over the vector sets $\{S, K\}$.

We can vectorize the expression "A hard working student with good grades is more likely than only a student with good grades" to see how the quantum model includes the fallacy. The hypothesis is over the word that causes the wrong state of belief in the probability assessment.

The projection of the state of belief Ψ that takes into account only students with good grades is produced in $S_y \cdot \Psi$ and the length is α. The probability of this is:

$$\Pr(S) = \left|S_y \cdot \Psi\right|^2 = |\alpha|^2$$

On the other hand, the projection of state of belief Ψ that takes into account at the same time hard work and the good grade is produced in $K_y \cdot \Psi$ and has the lenght β. The probability is:

$$\Pr(S \cap K) = \left|K_y \cdot \Psi\right|^2 = |\beta|^2$$

This model includes the *conjunction fallacy*: The likelihood of finding student with good grades is lower than that of finding students with good grades who are hard workers.

$$\Pr(S) < \Pr(S \cap K)$$

Third Component: The quantum analysis reveals the importance of order in the exercises

There is abundant literature about the influence of order in the types of exercises for which it respects the teaching as much as the own controls to evaluate the acquired knowledge. It is said that it affects more in the teaching than in the evaluations, although it always influences on the difficulty to determine the motive behind such variations.

It is important to emphasize that there exists an inherent difficulty in the study of the influence of order in the teaching, produced by the attention-distraction and activity-passiveness state of each student, both in the moment that the explanations occur as well as when the given exercises are solved by the teacher in class. As seen, the quantum models allow not to presume that state, and thus discover it later on as another variable in the analytics of the exercises. We will see below, in that same line, that the order also appears as an element but not assumed when the academic results are quantumly modeled, thus being able to solve the problems about the influence of order in the academic results.

Let's suppose two dichotomous teaching sets posed by the professor, which we call A and B, which can also be interchanged in the order that follows: if A then B, if B then A. After sequence A the grades obtained are analyzed, then sequence B is exposed and the grades are also analyzed.

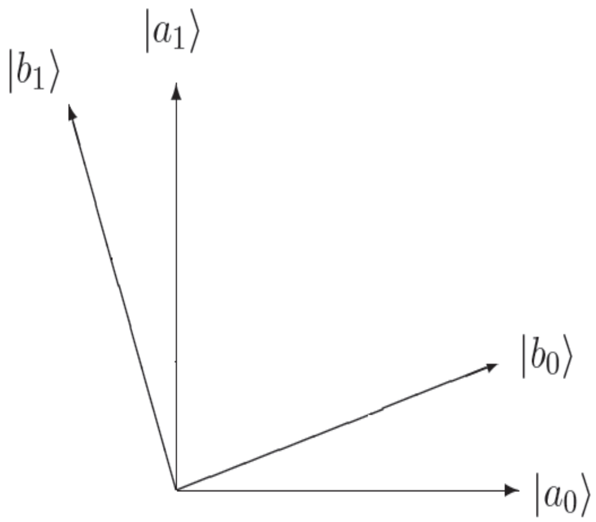
The question is the following: Being A the first teaching set, are better academic results being obtained?

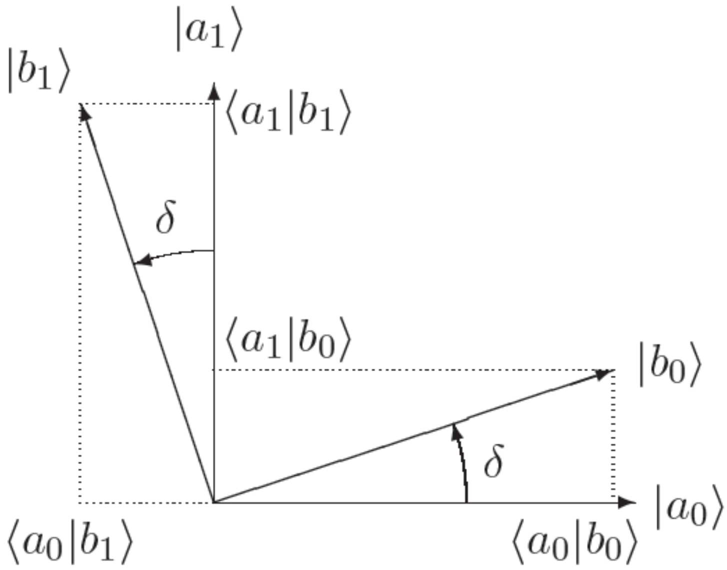
- If the answer is affirmative, it is represented by the vector $|a_1\rangle$
- Otherwise it is represented by the vector $|a_0\rangle$

In a similar way you can ask the following question: Being B the first teaching set, are better academic results being obtained?

- If the answer is affirmative, it is represented by the vector $|b_1\rangle$
- Otherwise it is represented by the vector $|b_0\rangle$

It is important to observe that the success of the teaching set is represented by a vector (or more exactly by the projection of said vector) and not by a plane or the subspace of a dimension greater than 1. Since there are no other possible answers for the sequences A and B (or B and A) other than 0 and 1, the set of vectors $(|a_0\rangle, |a_1\rangle)$ and $(|b_0\rangle, |b_1\rangle)$ form a basis of vector space of the possible answers and therefore the vector space is of a 2 dimension. In addition, the success or not of the teaching sets is produced in one same vector space, but with two different bases.





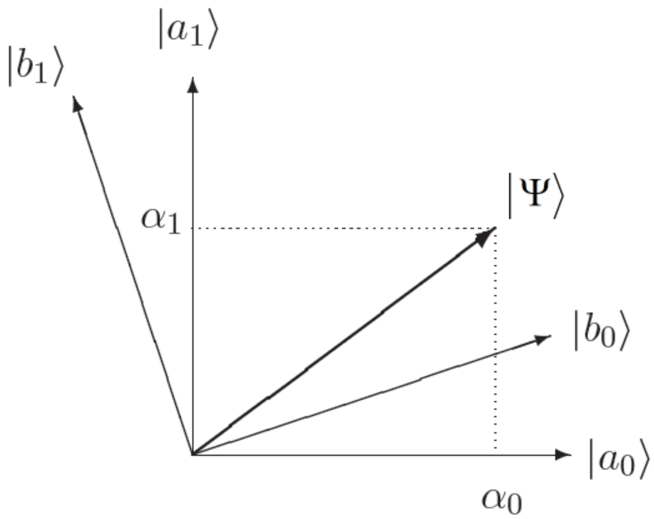
The vector basis $(|b_0\rangle, |b_1\rangle)$ can be decomposed in the other vector basis $(|a_0\rangle, |a_1\rangle)$:

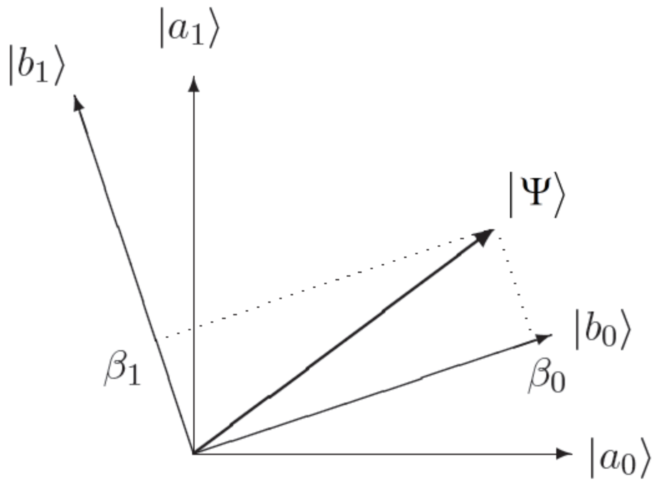
$$|b_0\rangle = \langle a_0, b_0\rangle |a_0\rangle + \langle a_1, b_0\rangle |a_1\rangle$$
$$|b_1\rangle = \langle a_0, b_1\rangle |a_0\rangle + \langle a_1, b_1\rangle |a_1\rangle$$

Likewise, the vector basis $(|a_0\rangle, |a_1\rangle)$ can be decomposed in $(|b_0\rangle, |b_1\rangle)$:

$$|a_0\rangle = \langle b_0, a_0\rangle |b_0\rangle + \langle b_1, a_0\rangle |b_1\rangle$$
$$|a_1\rangle = \langle b_0, a_1\rangle |b_0\rangle + \langle b_1, a_1\rangle |b_1\rangle$$

The scalar products are equal to $\cos\delta$ and $\sin\delta$.





The vector that represents the effect produced in the teaching set class is $|\Psi\rangle$ and can be expressed equally in the two orthonormal basis $(|a_0\rangle, |a_1\rangle)$ and $(|b_0\rangle, |b_1\rangle)$.

The last two vector representations assume the special case of a Hilbert space with real numbers, that is why it is supposed that it is equipped with the scalar product: for two vectors $|x\rangle$ and $|y\rangle$ the scalar product $\langle x|y\rangle$ is a complex number and its conjugate is $\langle y|x\rangle$.

The Hilbert space is about complex numbers and the vectors can be multiplied by any complex number.

Since the basis $(|a_0\rangle,|a_1\rangle)$ and $(|b_0\rangle,|b_1\rangle)$ are orthonormal:

$$\langle a_0,a_1\rangle=\langle b_0,b_1\rangle=0$$
$$\langle a_0,a_0\rangle=\langle a_1,a_1\rangle=\langle b_0,b_0\rangle=\langle b_1,b_1\rangle=1$$

As it has already been indicated, the cognitive status of the student is not presumed regarding the teaching sets A and B that they are working on. That is what the normalized vector $|\Psi\rangle$ represents and thus, as it happens in quantum mechanics, this modeling allows to collect predictive information about the learning achieved by the order of the teaching set.

$|\Psi\rangle$ can be represented in the basis $(|a_0\rangle,|a_1\rangle)$ as

$$|\Psi\rangle=\alpha_0|a_0\rangle+\alpha_1|a_1\rangle, \text{ with } (\alpha_0,\alpha_1)\in\mathbb{C}^2$$

$|\Psi\rangle$ can also be represented in the basis $(|b_0\rangle,|b_1\rangle)$ as

$$|\Psi\rangle=\beta_0|b_0\rangle+\beta_1|b_1\rangle, \text{ with } (\beta_0,\beta_1)\in\mathbb{C}^2$$

The cognitive state of the student $|\Psi\rangle$ is determined in a probabilistic manner on the success of the teaching set and it is only changed once the information is collected after conducting the exercises according to the following two rules:

1. The probability that a teaching set X $(X=A,B)$ has success on the cognitive state of a student x_i $(i=0,1)$ is given by the square module of the scalar product between $|\Psi\rangle$ y $|x_i\rangle$:

$$\Pr(x_i)=|\langle x_i|\Psi\rangle|^2$$

2. The cognitive state of the student in the function of success x_i obtained in the solving of the exercises of the second teaching set is the normalized projection of his cognitive state prior to the teaching set over the vector $|x_i\rangle$ according to the success achieved:

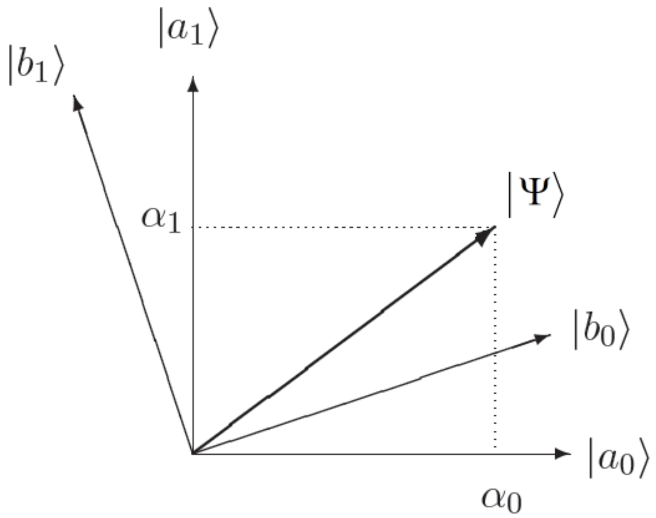
$$|\Psi\rangle\rightarrow\frac{\langle x_i|\Psi\rangle}{|\langle x_i|\Psi\rangle|}|x_i\rangle$$

For example, if the cognitive state of a student is described by the vector

$$|\Psi\rangle = \alpha_0 |a_0\rangle + \alpha_1 |a_1\rangle$$

the probability of having success 'i' the teaching set A is determined by $|\alpha_i|^2$ and the cognitive state after answering the exercises of the teaching set is:

$$\frac{\alpha_i}{|\alpha_i|}|a_i\rangle$$



In this graph the probability is obtained by projecting $|\Psi\rangle$ over the vector basis that corresponds to cognitive success, to then take the square of this length. If the success occurs (or not) with the set "if A then B", the state is projected over the basis $(|a_0\rangle, |a_1\rangle)$. If the success occurs (or not) with the set "if B then A", then the state must be projected over the basis $(|b_0\rangle, |b_1\rangle)$.

This type of quantum modeling binds both the learning of the student with the teaching success in function of the order of the exercises, considering the cognitive state before and after solving the teaching set, according to the probabilities projected in two references basis.

Conclusion

Despite more and more work being carried out, we still lack understanding on how metacognitive scaffolding affects learner's behaviour within complex problem resolutions. But particularly, up until now, we haven't been able to specify how a didactic that stresses student's reflection on their own knowledge would increase exercises results and, in general, their academic performance. This is the task which Quantitas is developing and the results up until now are very encouraging (In this link some scores).

By deepening the work of Kahneman, it has been discovered that the same kind of subjective heuristic structure which establishes the evaluation of benefits of decisions in relation to a reference point, is the same as that used in developing cognitive strategies that make knowledge acquisition easier. This means that the apparent irrationality that contradicts the laws of probability also underlies irregular temporal variations observed in the solving of exercises as they are reduced with praxis and error corrections.

It has also been observed that this 'subjectivity' could be related to Howard Gardner's categorization of intelligence profiles, above all analysing the relationship between temporal variations and kinematic-motor capacities. A scientific conclusion has not yet been established because massive data compilation, advanced mathematical modelling and cognitive computational systems are required to accelerate the verification and falsification processes of the set hypotheses. It could also have a remote relationship with the first motor and linguistic discoveries of childhood (from 9 months to 3 years old), but this research is beyond the scope of the current project.

Within the classroom, it has been shown that this heuristic subjectivity can be quickly improved by the students themselves when metacognitive information is included within the teaching process. In this way, the students are the ones that advance in an autonomous way within the strategies, in the same way that they started to walk or talk by themselves. This comparison is useful because kinematic provides lots of information about the cycles being observed in the sequences of attention-distraction and activity-passivity.

As a consequence, when analyzing irrationality within learning processes from the errors and the way they appear and disappear, it can be observed how each student evolves against the work of the teacher and at the same time that with this data they prepare a more precise heuristic didacticism to adapt to the new appearing metacognitive necessities.

When all this information has been categorized with objectivity, it can also be observed that the heuristics teacher-learner(s) feedback and didactics success grows significantly, above all in the long term. In other words, all of these factors combine to create a spontaneous and fruitful process in order to approach unique student 'reference points'. This is the case when the teaching process is full of useful metacognitive information that invigorates the learning process.

This new situation automatically allows for conceptualizations to be created and allows the development of superior metacognitive skills, making it easier in this way to solve non-structured exercises and above all allows for their adaptation to different students' profiles.

Finally, it has been found that quantum modelling allows us to analyze all this information because its principles do not assume mental states with respect to time, the grades obtained and the number of solved exercise.

Research design, scientific team to contact and method

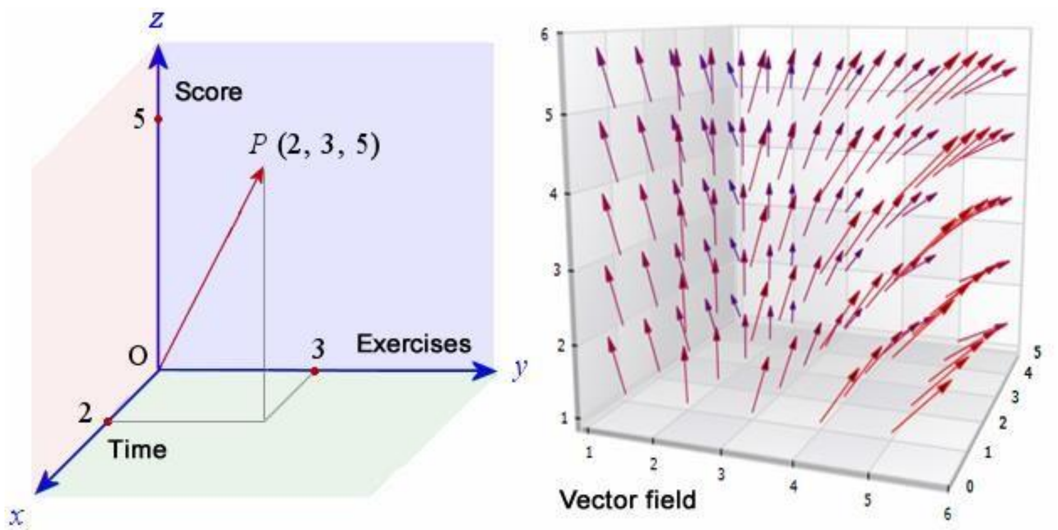
The aim of the research consists in building-up an automatic recommendation system of didactic sequences for the teacher in accordance with the detected students' profiles, compiling the pedagogic efforts from across all the classrooms. The purpose is to obtain an intelligent three-level system:

1. Prediction of marks and timing before starting the units of the topics, finding out which students are going to lack or exceed levels and where the average performance of the class may be set.
2. Prediction of the mark of the test to be done at the end of each unit.
3. Deduction of each student's intelligence profile, according to Gardner's analyses. More data will be needed here: corporal mobility and reading-writing capacities.

All of this would constitute the first tool that is able to learn from the students and teachers and thus through the accumulation of experience, **become a teachers' teacher and students' student**.

Design

To construct a predictive engine for Quantitas on the mathematical idea of 'vector field', understanding the didactic communication teacher-student schema in the class as a reflection of the magnetic field in an atomic nucleus, where the teacher is in the center and the students are moving in 'attention orbitals' accordingly to their intelligences' profiles.



To determine the vector's components, we will take as the departure point the previously explained quantum models with the purpose of evaluating the time lapse for solving activities, correcting the 'effort' in relation to the obtained mark and establishing didactic sequencing as well as the most suitable order within the exercises for each unit of the syllabus. Temporalization assessment is specifically important when understood as an interference of two attention-distraction waves.

Using Watson Analytics (IBM), to design a prediction based on two assumptions that must be corroborated for each unit of the syllabus:

1. Before each unit, the teacher loads the complete software package of the exercises he wishes to teach and the system makes a prediction regarding:
    a. Best distribution of the didactic sequences (scheduled in exercises/days), including a spiral of primary and secondary didactic sequences.
    b. Forecast of marks and timing of each exercise for each student.
    c. Workload for the students, arranging the class in three different blocks:
        i. Average performance of the class, automatically monitored by the system.
        ii. Students who need more time. What exercises and remedial work are needed to overcome difficulties.
        iii. Students who have got spare time because the syllabus does not fulfil their expectations. They face a schedule of superior difficulty exercises and individual strategies.
    d. Metacognitive evaluations regarding:
        i. The 'reference point' of each student.
        ii. Potential evolution of their 'cognitive shortcuts'.
2. When finishing the unit, the teacher loads the final test and the system makes a prediction about the qualifications and the needed time to completely solve it.

The predictive system is constituted so that it can learn by itself:

1. Within the first prediction, the system can corroborate its own precision according to the data inputted from the students' exercises, whilst the didactic units continue on
2. Within the second prediction, the system verifies by means of the test results

In addition, a kinematic-motor assessment will be introduced by means of a frame per second recording of the mobility of the students within the classroom, plus their fingerprints on the tablet's reader. The objective here is to determine the general mobility of the body and the accuracy of the hand in order to establish a relationship between intellectual and kinematic-motor processes.

It employs a circular development schema, together with the following two analytical stages:

Before the test, during the unit:
1. Prediction of the time for each exercise per student, in relation to kinematics.
2. Determination of the 'intelligence orbital'
3. Initial assessment of necessary 'effort'
4. Individualized didactic sequences and workload design
5. Establishment of a hypothetic 'vector' for each exercise-student pair
6. Real-time reception of the results achieved within the classroom.
7. Corroboration of the previous hypotheses, valuations and predictions.
8. Correction of the predictive model.

For the test:
1. Prediction of the time and mark for each exercise-student, concerning the received feedback of the work of each student before the test.
2. Reception of the results of the test.
3. Corroboration of the prediction.
4. Correction of the predictive model

Method

3-year work cycle, with 300 centers in 3 countries (100 each country)

Year	Spain	US	Israel
I	10		
II	100	100	100
III	100	100	100

1. Spin-off development, 'start-up' style
2. Distributed working system, like kernel SO Linux
3. Ongoing communication between teachers and scientists, type GitHub and Stack Overflow
4. Shared didactic experiences among students of different nationalities
5. Government administration and PISA-OCDE implication.
6. Public visualization of results on the Internet with map.
7. Ongoing communication of the project to the rest of the countries and Autonomous Communities.
8. Valuation of new candidates for project expansion.

Procedure and data treatment by means of a first meeting in Euskadi of everyone involved in the team, the basic guidelines will be established to monitor throughout the first year the obtained results in the ten first pilot centers.

At the same time data is being collected and at the end of the academic year, communications between the teams will be established and in a second meeting a definitive proposal of scientific research will be completed for the following two academic years in the mentioned countries.

Exercise typology to be introduced into the system

- Level 1: Monotonous, basic and methodical, to reveal implicit metacognition.
- Level 2: Non-monotonous, complex and methodical, to sort out metacognitive levels.
- Level 3: Non-monotonous metacognitive exercises, complex and unstructured, sorted by the 11 types previously mentioned. Achieving this level within the parts of the syllabus that allow for this didactic objective of Quantitas.

quαntitas